Application of Digital Stochastic Measurement of Definite Integral Product of Two or More Signals Using Two-Bit A/D Converter

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***Abstract* — Commonly used strategy in discrete digital measurements is to capture digital value of signal's magnitude at a chosen time instant. The mathematics in the background of this strategy is algebra, while the applied theory is the Theory of discrete signals and systems. An alternative measurement strategy, named "measurement over an interval", has been researched in three challenging areas: (i) measurement of fast-changing signals, (ii) measurement of noisy signals, and (iii) measurement that requires high accuracy and linearity. Numerous simulations, experiments and developed measurement instruments have proven the engineering/metrological applicability of this "measurement over an interval" strategy. This paper presents application of digital stochastic measurement over interval of the finite integral product of two or more signals using two-bit A/D converter. Error of this method is shown through a large number of simulations.**

Keywords *– A/D conversion, Digital measurements, Probability, Stochastic processes.*

I. INTRODUCTION

Nowadays, the term „measurement" is mostly considered as discrete digital measurement (sampling method measurement). This commonly used strategy called "measurement in a point" is to capture digital value of signal's magnitude at a chosen time instant. Digital representation of the signal's parameters at that time instant has to be characterized with maximum accuracy (theoretically - with an ideal accuracy). In order to capture full information on the measured quantity, all conditions of the sampling theory must be satisfied. The mathematics that explains this approach is algebra, while the applied theory is the Theory of discrete signals and systems. This measurement strategy has been the backbone of the development in metrology, control, telecommunications, etc. In the conversion process, accuracy and speed are opposing requirements. Accurate measurements of low-level, noisy and distorted signals have been a challenging problem in the theory and practice of measurement science and technology. Since 1956 [1], the possibility for reliable operation of instruments with inherent random error has been researched. It has been shown that adding a random uniform dither to the input signal prior quantization, decouples the measurement error from the input signal [2].

Measurement strategy "measurement over an interval" formulated in [3] has clear advantages over standard "measurement in a point" approach in three challenging areas:

(*i*) measurement of fast-changing signals,

(*ii*) measurement of noisy signals,

(*iii*) measurement that requires high accuracy and linearity.

The third case, which is of the greatest importance in metrology, is elaborated in [4]. Measurement over an interval entails a very simple hardware (flash A/D converter as the simplest and the fastest, but the least precise device) hence lowering the number of systematic error sources (it is well known that the flash A/D converter hardware is being doubled with each new bit of the resolution thus doubling a systematic error sources). The sampling method has two inherent sources of systematic measurement error: discretization in time and discretization in value. If the sampling theorem conditions are satisfied, discretization in time is eliminated as a source of measurement error. Discretization in value always causes a systematic measurement error – it cannot be eliminated. In [4] it is shown, under certain conditions, how to reduce it and keep within acceptable range.

Measurement over an interval is an integral approach to measure a signal and its parameters - a signal [5] or some of its parameters [4] are measured during a finite time interval of an arbitrary duration. It has been shown that a singular measurement in every instant does not need to be maximally accurate, while the measurement uncertainty is reduced by adding a random uniform dither. Mathematical tools utilized in this case belong to mathematical analysis, probability theory, statistics and sampling theory. From theoretical point of view, the problem is highly non-linear and stochastic, and therefore neither the standard linear Theory of discrete signals and systems nor the Theory of random processes can be applied. It was necessary to develop an alternative mathematical approach, and this approach has been developed based on Central limit theorem. The most commonly we use measurement devices in greed measurement: measurement of true RMS of current and voltage, as well as power and energy, also with some derived values is taking place (cos φ, distortion factor...).

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II. Principle of Measurement

Fig. 1 shows the principle of adding a uniform random dither h to the measured signal y. The role of the dither is to decouple the quantization error of the uniform quantizer from the input signal [2], and thus enabling accurate measurements using a low-resolution converter [4]. Ψ is output of the uniform quantizer of the digital multiplier.

Figure 1. Block diagram of application of a uniform random dither h to the measured signal y.

For the subsequent discussion let us assume that the following conditions are satisfied:

$$|y| \le R,\ R = Z \cdot a,\ |h| \le \frac{a}{2},\ |y+h| \le R + \frac{a}{2} \qquad (1)$$

where range of the input quantity is labeled with R, number of positive quantization levels with Z and quantum of uniform quantizer with a. A probability density function (PDF) of uniform random distribution dither signal h is $p(h) = \frac{1}{a}$ for $|h| \le \frac{a}{2}$. We can interpret it graphically as in Fig. 2.

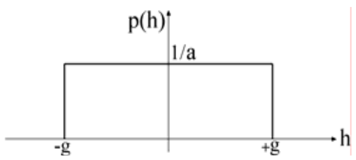


Figure 2. PDF of the uniform random dither signal h.

Voltage ranges and decision thresholds associated with process of measuring average input signal by uniform quantizer are represented graphically in Fig. 3:

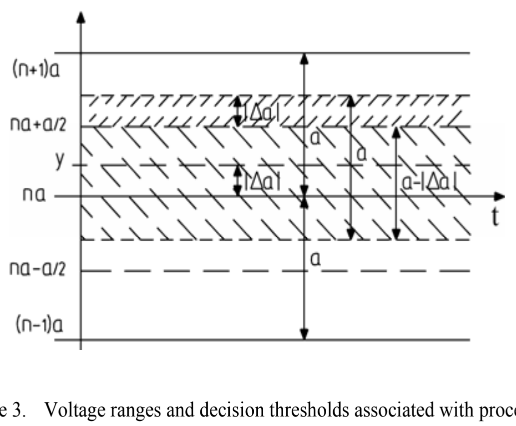


Figure 3. Voltage ranges and decision thresholds associated with process of measuring $\overline{\Psi}$.

For $Z = 1$, the A/D converter is very simple, with the minimal hardware structure as shown in Fig. 4.

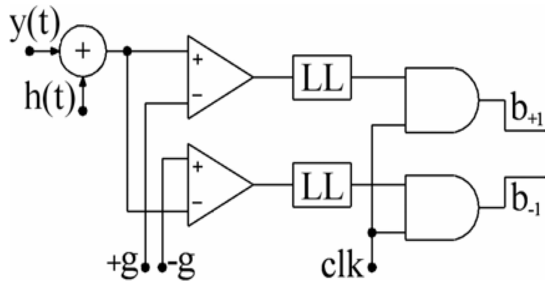


Figure 4. Two-bit flash A/D converter with an additive uniform dither.

For the device in Fig. 4 the quantum $a = 2g$ i.e.

$$g = \frac{a}{2},\quad R = a = 2g,\quad p(h) = \frac{1}{2g} \qquad (2)$$

Possible values of Ψ are $\Psi \in \{-2g, 0, 2g\}$, and the analytical term for Ψ is:

$$\Psi = 2g \cdot (b_1 - b_{-1}) \qquad (3)$$

where $b_1, b_{-1} \in \{0,1\}$ and $b_1 \cdot b_{-1} = 0$. It is never possible that b_1 and b_{-1} are equal to 1 simultaneously - it would have meant that $y \ge 0$ and $y \le 0$ simultaneously.

III. Digital Stochastic Measurement over Interval of the Finite Integral of Two Signals Product

Device from Fig. 5 has 2 two-bit flash A/D converters shown on Fig. 4, with inputs $y_1 = f_1(t)$ and its uncorrelated signal h_1, as well as $y_2 = f_2(t)$ and its uncorrelated signal h_2, respectively. h_1 and h_2 are mutually uncorrelated random uniform dither signals. Outputs Ψ_1 and Ψ_2 are passed to a multiplier; the multiplier output is $\Psi = \Psi_1 \cdot \Psi_2$, and it can assume values: $\Psi \in \{-(2g)^2, 0, +(2g)^2\}$.

If, during one measurement interval, N A/D conversions are performed by each A/D converter, then the accumulator from Fig. 5 accumulates the sum of N subsequent multiplier's outputs: $\sum_{i=1}^{N} \Psi_1(i) \cdot \Psi_2(i)$. This accumulation can be simply used for calculation of the average value of the multiplier output $\overline{\Psi}$ over the measurement interval as:

$$\overline{\Psi} = \frac{1}{N} \cdot \sum_{i=1}^{N} (\Psi_1(i) \cdot \Psi_2(i)) \qquad (4)$$

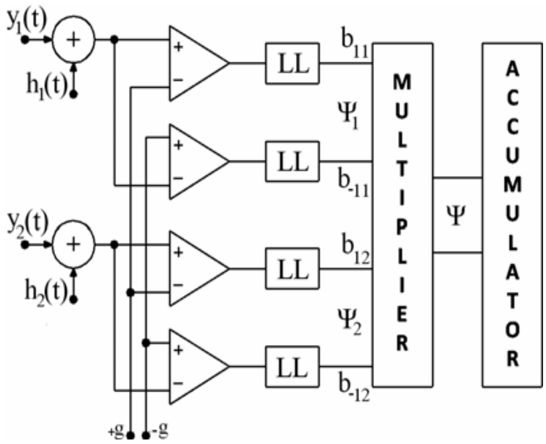


Figure 5. Structure of the device, based on two-bit flash A/D converters, for measurement of the finite integral. of two signals product

For Measurement purpose real input signals are being conditioned and thus reduced to appropriate voltage level ±5 V (U_{max} = +5 V, U_{min} = -5 V). On that input signal a random voltage signal is now superimposed. Random voltage signal is dieter with level ±2.5 V.

A. Application in the measurement of true RMS

To analyze measurement of signal true RMS we will observe two wave forms: simply periodic signal (sinusoidal signal with frequency 50 Hz) and typical complex periodic signal (jagged signal with same frequency).

Simulation is done with signal amplitude of 0.5 V, 1 V, 1.5 V, 2 V, 2.5 V, 3 V, 3.5 V, 4 V, 4.5 V and 5 V, respectively. For each amplitude of the input signal we perform a series of 100 measurements with different duration: 20 ms, 40 ms, 60 ms, 80 ms, 100 ms, 200 ms, 500 ms, 1 s, 2 s, 5 s, 10 s, 20 s, 30 s, 1 min, 2 min, 5 min, 10 min and 15 min respectively.

To calculate true RMS of individual measurement it is necessary to multiply U_{max} with the square root of the mean output. Now we can start calculating individual measurement errors as:

- absolute error = measured value - true value
- relative error = absolute error / true value · 100%
- errors relative to full scale (FS) = absolute error / ($U_{max} - U_{min}$) · 100%

After we do these calculations for each 100 measurements group we calculate mean and standard deviation for both relative error and error relative to FS (absolute error does not give us any important information about the accuracy nor precision of measurement).

Fig. 6 shows error diagrams for measurements of true RMS of simply periodic (sinusoidal) signal for error in relation to the FS.

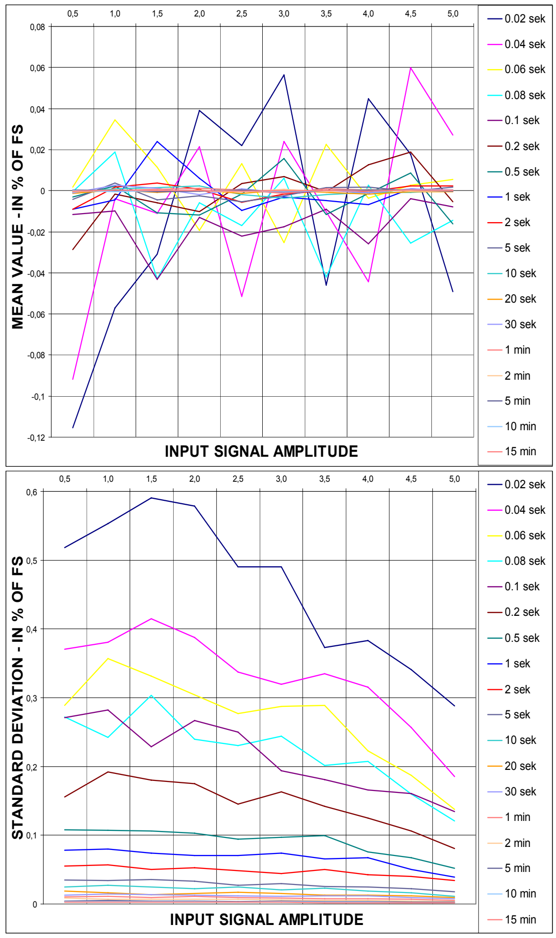


Figure 6. Measurement error of true RMS of simply periodic signal for error in relation of FS.

The theoretical error [6] is presented by expression:

$$\sigma_e^2 = \frac{(2g)^2}{t_2 - t_1}\cdot\int_{t_1}^{t_2}\left|f_1(t)f_2(t)\right|dt - \frac{1}{t_2 - t_1}\cdot\int_{t_1}^{t_2} f_1(t)^2 f_2(t)^2\,dt \qquad (5)$$

Simulation results confirm theoretically expected error. It is important to note that this method requires accurate voltages of two thresholds (+ 2.5V and -2,5V) and uniform dither signal distribution. If we compare this method with classical digital measurement approach, it can be calculated that 14-bit A/D converter should be used in digital measurement approach for obtaining the error of presented method.

Fig. 7 shows error diagrams for measurement of true RMS of complex periodic (jagged) signal for error in relation to the FS.

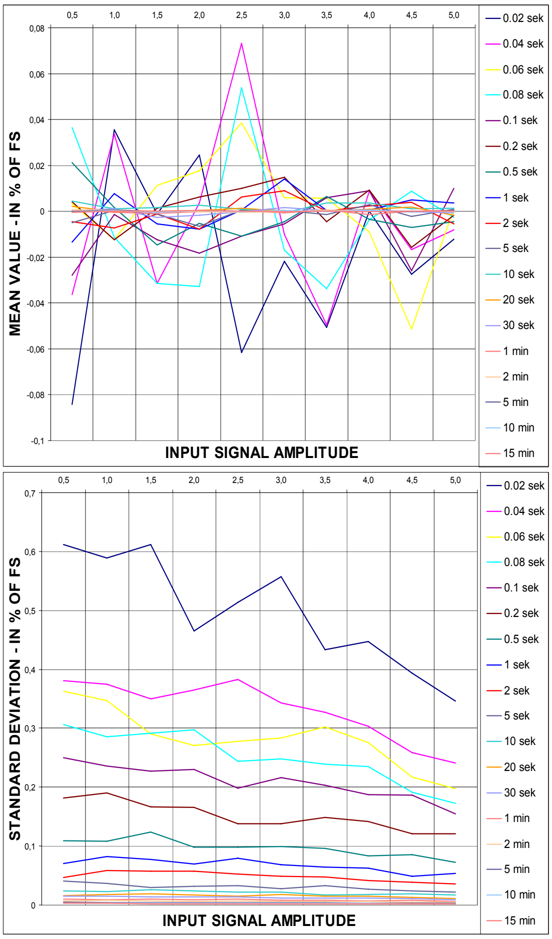


Figure 7. Measurement error of true RMS of complex periodic signal for error in relation of FS.

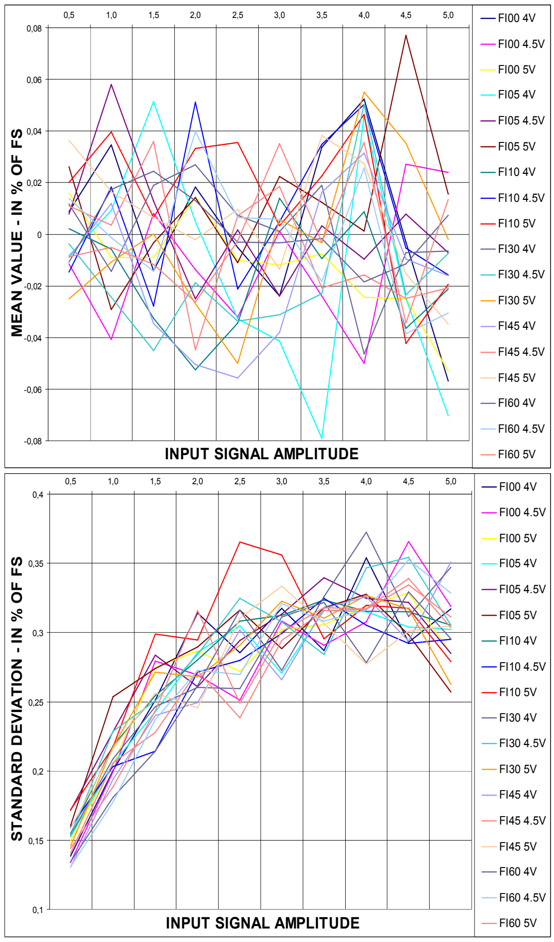


Figure 8. Measurement error of power of simply periodic signal relative to the FS.

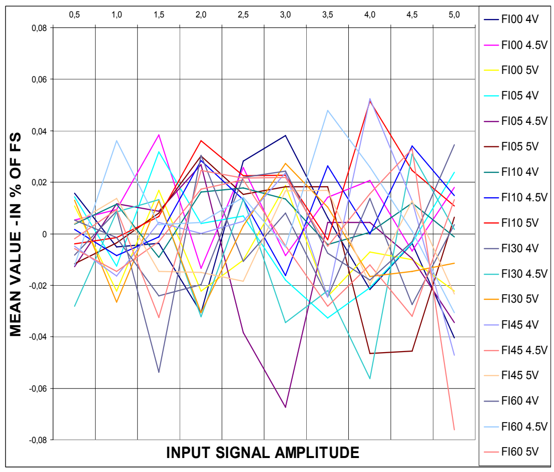
B. *Application in the measurement of power*

In the analysis of the signal strength measurement will observe the same waveforms as in the measurement of the true RMS. Unlike the measurement of the true RMS where we had one input, the strength measurement we do on two input signals. One of them represents the voltage signal, and the other one represents a current signal. In grid measurements voltage signal is approximately constant therefore we try to scale it in the "upper zone" in order to have less measurement error, so the simulation is also done for the amplitude of 4 V, 4.5 V, and 5 V. For each of these signals conditioned current signal takes a value of 0.5 V, 1 V, 1.5 V, 2 V, 2.5 V, 3 V, 3.5 V, 4 V, 4.5 V and 5 V respectively. In addition for each combination of power measurements we do simulation for phase shifts of π/36 (5°), π/18 (10°), π/6 (30°), π/4 (45°), π/3 (60°). The duration of each measurement is 1 s.

To calculate power of individual measurement it is necessary to multiply U_{max}^2 with the mean output. Now we can start calculate individual measurement errors like we have done in the measurement of the true RMS.

Fig. 8 shows error diagrams for measurements of power of simply periodic (sinusoidal) signal for error relative to the FS.

Fig. 9 shows error diagrams for measurement of power of complex periodic (jagged) signal for error relative to the FS.



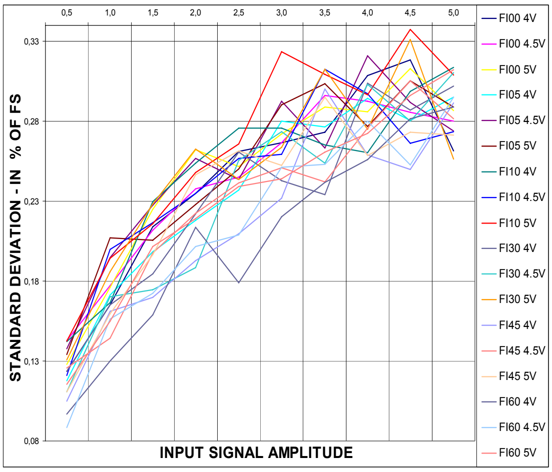


Figure 9. Measurement error of power of complex periodic signal relative to the FS.

IV. Digital Stochastic Measurement over Interval of the Finite Integral of More than Two Signals Product

In all so far published articles, of which some are listed in references [3-6] it is processed application of devices on 2 signal product. Each of the articles includes the theoretical part. Some articles include simulation and description of the prototype device. However neither article describes application for product of more than two signals beside theoretical conclusions. To show a practical implementation of the method for the measurement of the product of more than 2 signals, that is until now only mentioned as a possibility for the generalization of the method, we give an example of measuring the power of the wind. Measurement of wind power [7] is calculated by the formula:

$$P_S = 0{,}5 \times \rho \times A \times v^3 \qquad (6)$$

wherein:

ρ – density of air (1,25kg / m3)

A – surface area through which air flows

v - air velocity

This formula is in practice reduced to a formula in which besides the speed of wind flow exists only circle diameter (D) that windmill wings plot:

$$P_S = 0{,}291 \times D^2 \times v^3 \qquad (7)$$

As you can see, wind power is a value proportional to the cube of its speed. Measurement of wind speed itself is done with an anemometer [8]. The measurements are performed on different locations in quite long period (up to one year). After that we determine the best place to set up a windmill and its optimal position for maximum efficiency at a chosen location.

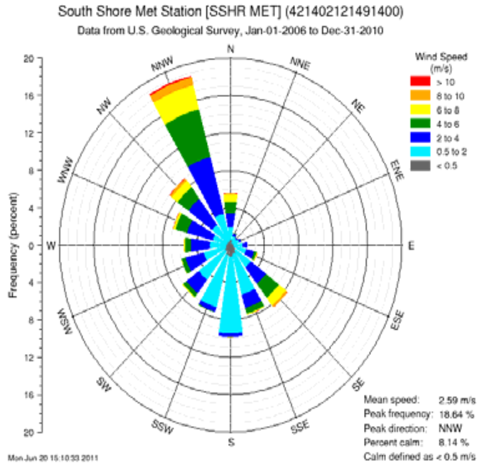


Figure 10. Wind rose [9].

Important parameters are the wind rose and the relative frequency of the wind histogram. Fig. 10 shows the direction of the wind in the period.

Fig. 11 shows the relative frequency histogram from where you can see how long each wind blows. These two parameters are important for determining the viability of a potential investment.

For this purpose, the device in Fig. 5 is being extend, as shown in Fig. 12. Device from Fig. 12 has 3 two-bit flash A/D converters from Fig. 4, with inputs $y_1 = f_1(t)$ and signal h_1, and $y_2 = f_2(t)$ and signal h_2, and $y_3 = f_3(t)$ and signal h_3, respectively. h_1, h_2 and h_3 are mutually uncorrelated random uniform dither signals. In this special case is $y_1 = y_2 = y_3 = v(t)$. Outputs Ψ_1, Ψ_2 and Ψ_3 are passed to a multiplier; the multiplier output is $\Psi = \Psi_1 \cdot \Psi_2 \cdot \Psi_3$, and it can assume values: $\Psi \in \{-(2g)^3, 0, +(2g)^3\}$.

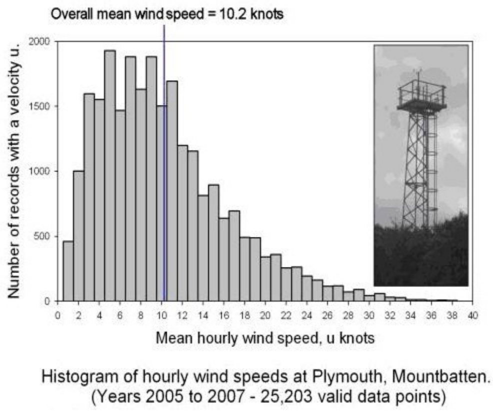


Figure 11. Histogram of hourly wind speed [10].

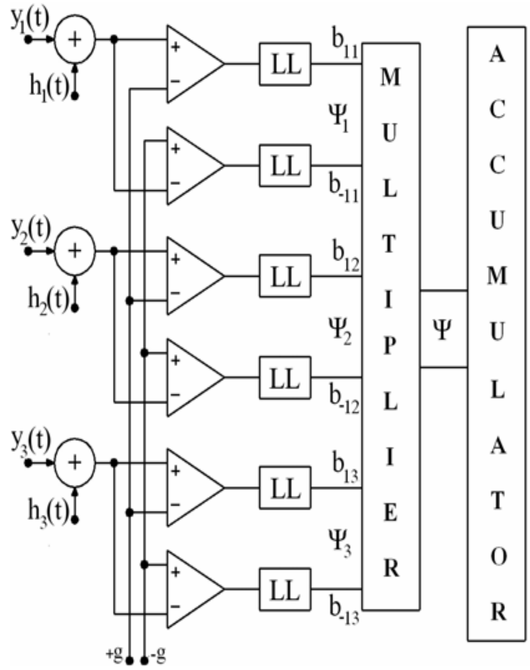


Figure 12. Structure of the device, based on two-bit flash A/D converters, for measurement of the finite integral product of three signals.

The corresponding simulation was performed to measure wind power with data usage from the website [11] for June, 11th 2011. The sampling frequency of 7.6 Hz was used, and data amount is 656104. Explanation of presented data is given on the website [12]. Maximum wind speeds at a given location are up to 25 m/s, while the particular sample maximum speed is 23.89 m/s. It is necessary to condition input signal (as well as the signals of current and voltage used in the previous sections) in order to obtain a voltage signal of an appropriate level (± 5 V).

To calculate wind energy of presented data it is necessary to multiply U_{max}^3 with the output value. To calculate wind power of presented data it is necessary to multiply U_{max}^3 with the mean output. Now we can start calculate individual measurement errors like we have done in both the measurement of the true RMS and the measurement of the power.

For a given set of data the measured energy by using that device is $148.5 \cdot D^2$ MJ, while the average power is $1720 \cdot D^2$ W. The corresponding errors are: absolute error = $3.83 \cdot D^2$, relative error = 0.223 % and errors relative to FS = 0.112 %

V. Conclusion

This paper presents the possibilities of digital stochastic measurement methods i.e. application of measurements of the definite integral product of two or more signals using two-bit flash A/D converters. Most important idea of this approach is to treat the time within the integration interval as an independent uniform random variable. In the developed method no assumptions are made regarding the waveform shape of measured signals. The two-bit flash A/D converter's design is rather simple, making it suitable for simple measurement of signal, and for parallel measurements without affecting conversion speed, measurement precision and accuracy.

Measurement cases of true RMS and power for both simple periodic (sinusoidal) and complex periodic wave form of signal we use for measuring of two signal product integral were examined trough simulation. In addition, the simulation was performed to measure three signal product, for which we show an example in measurement of wind power (wind power is proportional to the wind speed cube). Simulation samples are real life data with a sampling frequency of 7.6 Hz. Simulations confirm the theoretical expectations.

References

[1] J. von Neumann, “Probabilistic logic and the synthesis of reliable organisms from unreliable components,” in *Automata Studies*, C. E. Shannon, Ed. Princeton, NJ: Princeton Univ. Press, 1956.

[2] M.F. Wagdy and W. Ng, "Validity of uniform quantization error model for sinusoidal signals without and with dither." *IEEE Transactions on Instrumentation and Measurement*, vol. 38, no. 3, June 1989. pp.718-722, doi: 10.1109/19.32180.

[3] V.V. Vujicic, I. Zupunski, Z. Mitrovic and M.A. Sokola, "Measurement in a point versus measurement over an interval." *Proc. of the IMEKO XIX World Congress*; Lisbon, Portugal. Sep. 2009. pp. 1128-1132 no.480.

[4] D. Pejic, M. Urekar, V. Vujicic and S. Avramov-Zamurovic, "Comparator offset error suppression in stochastic converters used in a watt-hour meter.", in *Proc. CPEM 2010, Proceedings*; Korea. June 2010.

[5] V. Pjevalica and V.V. Vujicic, "Further generalization of the low-frequency true-RMS Instrument." *IEEE Transactions on Instrumentation and Measurement*, vol. 59, no. 3, March 2010. pp. 736-744.

[6] B. Ličina, P. Sovilj, Application of Digital Stochastic Measurement over an Interval in Time and Frequency Domain, pp. 297-302, International Conference ICIST 2014, Kopaonik, 09.03.–13.03.2014.

[7] M. Milosavljevic, M. Marjanovic, M. Obućina, "The estimation of wind energy potential for the production of electrical energy in Petrovac na Mlavi", in 56th *Proc. ETRAN Conference, Zlatibor, ML3.2-1-4, June 11-14, 2012.*

[8] M. Zlatanovic, "What does a cup anemometer measure?", in 56th *Proc. ETRAN Conference, Zlatibor, ML3.1-1-4, June 11-14*, 2014.

[9] http://nj.usgs.gov/grapher/tutorial/graphs/example_wind_rose.png.

[10] http://www.wind-power-program.com/wind_statistics.htm.

[11] https://code.google.com/p/google-rec-csp/downloads/detail?name=06-11-2011.txt.gz&can=2&q=&sort=-size.

[12] http://google.org/pdfs/google_heliostat_wind_data_collection.pdf.