Design of Two-Channel Analysis Part of Hybrid Filter Bank

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***Abstract*—In the present paper a new design method for continuous-time power-symmetric active RC filters, which is suitable for two-channel hybrid filter bank realization, is proposed. Some theoretical properties of continuous-time power-symmetric filters in a more general perspective are studied. This includes the derivation of a new general analytical form, and a study of poles and zeros locations in s-plane. In the proposed design method the analytic solution of filter coefficients is solved in s-domain using only one nonlinear equation. Finally, the proposed approximation is compared to standard approximations. It was shown that attenuation and group delay characteristic of the proposed filter lie between Butterworth and elliptic characteristics.**

Keywords- Hybrid filter bank, power complementary filter pair, rational transfer function, allpass network, active RC filter.

I. Introduction

Many continuous-time signals have a low level nature, such as the output of sensors, then their processing often require multi-band decomposition for time-frequency analysis, manipulation, recognition of the signal, or storage. The hybrid filter bank (HFB) can be used for these applications and it is also suitable for high resolution conversion between analog and digital signals. Therefore, HFB is associated with analog to ADCs working at lower sample rate in comparison with the Nyquist sampling rate. Thus, the HFB is an unconventional class of the filter bank that employs both analog and digital filters [1]–[4].

The principle of a continuous-time linear hybrid two channel filter bank is shown in Fig. 1. The system consists of an continuous-time analysis two channel filter bank, uniform samplers, quantizers, and a discrete-time synthesis filter bank. The analysis filter bank consists of the low-pass filter $H_0(s)$ and high-pass filter $H_1(s)$. Both filters have the same passband edge and split the spectrum of the band limited input signal $x(t)$ by the factor of 2. The sampling and quantization takes place at the output of the analysis filters with the twice lower sampling frequency $1/(2T_s)$. The quantized signal goes into a linear discrete-time synthesis filter bank, which generate a single signal from two upsampled and interpolated signals. The up samplers are used to retain the desired Nyquist sampling rate $1/T_s$.

The continuous-time filters chosen to build the analysis filter bank play an important role in the performance of the hybrid filter bank. It is known that continuous-time filters of odd degree can be realized as the sum of two stable all-pass filters with real coefficients having no common poles [5]. As all-pass sums, such filter bank can be realized with low complexity structures that are robust to finite precision of components.

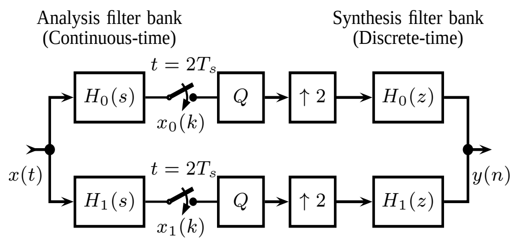

Fig. 1. Two channel hybrid continuous/discrte filter bank.

For the definition purposes, consider a continual prototype lowpass-highpass filter pair, denoted by $[H_0(s),\ H_1(s)]$, where $H_0(s)$ is transfer function of lowpass part of filter pair and $H_1(s) = H_0(s^{-1})$ is transfer function of highpass part filter pair. Normalized passband edge for both filters is equal to one.

A filter pair $[H_0(s),\ H_1(s)]$ is a *power-complementary* filter pair [6] if the sum of the squares of their magnitude responses satisfies

$$H_0(s)H_0(-s) + H_1(s)H_1(-s) = 1 \tag{1}$$

or at real frequency $s = j\,\omega$

$$|H_0(j\omega)|^2 + |H_1(j\omega)|^2 = 1 \tag{2}$$

For this pair, the angular frequency $\omega_c = 1$, where $|H_0(j)|^2 = |H_1(j)|^2 = 0.5$, is the *crossover angular frequency*. At this angular frequency, the gain responses of both filters are approximately 3 dB below their maximum values. Note, crossover angular frequency is 3 dB passband edge for both lowpass and highpass part of filter pair. Thus, $H_1(s) = H_0(s^{-1})$.

A filter pair $[H_0(s),\ H_1(s)]$ is an *all-pass-complementary* filter pair [7] if the sum and difference of $H_0(s)$ and $H_1(s)$ satisfies

$$\begin{aligned} H_0(s) + H_1(s) &= A_1(s) \\ H_0(s) - H_1(s) &= A_2(s) \end{aligned} \tag{3}$$

where $A_1(s)$ and $A_2(s)$ are all-pass transfer functions.

Transfer function sets which are simultaneously all-pass complementary and power complementary are termed *double complementary*. All double complementary filter pair can be expressed as the sum of stable allpass filters such as the Butterworth and the elliptic filters. The other classical approximation cannot form a double complementary pair.

New efficient approximation of the doubly complementary filter pair is proposed in this paper. The realization, based on the continuous-time all-pass filters, which validate this approach is also presented.

II. Approximation

Necessary and sufficient conditions for the transfer function to be suitable for the realization of the continuous-time two channel filter banks are given in this section. In general, the squared magnitude characteristic of the lowpass prototype in the s-plane is expressed in the form

$$H_0(s)H_0(-s) = \frac{1}{1+K(s)K(-s)} \tag{4}$$

where filter characteristic function $K(s)$ is rational, and the polynomial in denominator contains only even or odd power of s, but, the polynomial in nominator contains only even power of s. For power symmetric filter design, at normalized passband edge frequency $s = \pm j$ the characteristic function is equal to one, then the insertion loss of filter at this frequency is 3.0103 dB. In fact, Butterworth, Chebyshev1, Chebyshev2, and elliptic filters are introduced in this form, and the filter properties are governed in a way where $K(s)$ is chosen [8].

Lemma 1: The rational transfer functions $H_0(s)$ and $H_1(s)$ in Eq. (1) satisfied power symmetric in z [9] and in s-domain if

$$K(s)K(-s)K(s^{-1})K(-s^{-1}) = 1 \tag{5}$$

Proof: For Eq. (4) we have

$$\begin{aligned} &H_0(s)H_0(-s) + H_0(s^{-1})H_0(-s^{-1}) \\ &= \frac{1}{1+K(s)K(-s)} + \frac{1}{1+K(s^{-1})K(-s^{-1})} \\ &= \frac{1+R(s)}{R(s)+K(s)K(-s)K(s^{-1})K(-s^{-1})} \end{aligned} \tag{6}$$

where $R(s) = 1 + K(s)K(-s) + K(s^{-1})K(-s^{-1})$. Clearly, this is equal to one if and only if

$$K(s)K(-s)K(s^{-1})K(-s^{-1}) = 1$$

This lemma is proved.

Based on the preceding result it is possible to develop a general analytic form for $K(s)$ which is suitable for continuous-time power symmetric filter design.

Lemma 2: A rational filter transfer function (4) satisfied power symmetric [10] in s-domain (5) if and only if characteristic function has the form

$$K(s) = s^k \prod_{m=1}^{M} \left(\frac{s^2+\omega_m^2}{\omega_m^2 s^2+1}\right)^{l_m} \tag{7}$$

with $\omega_m < 1$ and arbitrary integer l_m, for $m = 1, 2, \ldots, M$. Filter order is $N = k + 2\sum_{m=1}^{M} l_m$.

This condition can be expressed equivalently by

$$K(-s)K(s^{-1}) = (-1)^k \tag{8}$$

for all s.

Proof: This comes from the following facts:

1) $K(-s) = (-1)^k K(s)$
2) $K(s^{-1}) = \dfrac{1}{K(s)}$

This lemma is proved.

Note, for $k > 1$ and $l_1 = l_2 = \cdots = l_M = 0$ we have Butterworth filter which is power-symmetric. For $k = 0$ or 1 and $l_1 = l_2 = \cdots = l_M = 1$ we have Elliptic filters which are also power-symmetric. Chebyshev filters cannot be power-symmetric because they have ripples only in the passband or stopband.

Lemma 3: Let $H_0(s)$ be a rational with real coefficients power symmetric filter function, then all poles of it are restricted to be on the unit circle.

Proof: Since $K(s)$ has the form (7), its poles are restricted to be on the imaginary axis, then power symmetric $H_0(s)$ implies (8) and equation (4) can be rewritten as

$$H_0(s)H_0(-s) = \frac{1}{1+(-1)^k \dfrac{K(s)}{K(s^{-1})}} \tag{9}$$

At pole frequencies of $H(s)$ the denominator of the expression (9) is zero, that is

$$\frac{K(s)}{K(s^{-1})} = (-1)^{k+1} \tag{10}$$

In view of the real-coefficient assumption at $s = e^{-j\theta}$ we have $K(e^{-j\theta}) = K^*(e^{j\theta})$. On the unit circle of the s-plane we therefore have

$$\left|\frac{K(e^{j\theta})}{K(e^{-j\theta})}\right| = 1 \tag{11}$$

So, the quantity $K(s)/K(s^{-1})$ has unit-magnitude on the unit circle, then all poles of $H(s)H(-s)$ are on the unit circle.

Lemma 4: A filter pair $[H_0(s), H_1(s)]$ is an all-pass-complementary filter pair of transfer functions, i.e., a par satisfied

$$H_0(s) + H_1(s) = A_1(s) \tag{12}$$

where $A_1(s)$ is an stable all-pass transfer function. Then, the following equation is also automatically satisfied

$$H_0(s) - H_1(s) = A_2(s) \tag{13}$$

Proof: Since $A_1(s)$ and $A_2(s)$ are the allpass transfer functions, then $A_1(s)A_1(-s) = 1$ and $A_2(s)A_2(-s) = 1$. Further, the squared magnitude characteristic of left side of equation (12) is

$$\begin{aligned} G(s) &= [H_0(s)+H_1(s)][H_0(-s)+H_1(-s)] \\ &= 1 + H_0(s)H_1(-s) + H_0(-s)H_1(s) \end{aligned} \tag{14}$$

because filter pair $[H_0(s),\, H_1(s)]$ is power complementary.

Let N be odd. Transfer function $H_0(s)$ and $H_1(s)$ have the form

$$
\begin{aligned}
H_0(s) &= \frac{\prod_{m=1}^{M} (\omega_m^2 s^2 + 1)^{l_m}}{(s+1)(s^{N-1} + \alpha_1 s^{N-2} + \cdots + \alpha_1 s + 1)} \\
H_1(s) &= \frac{s^{N-2\nu} \prod_{m=1}^{M} (s^2 + \omega_m^2)^{l_m}}{(s+1)(s^{N-1} + \alpha_1 s^{N-2} + \cdots + \alpha_1 s + 1)}
\end{aligned}
\tag{15}
$$

where $\nu = \sum_{m=1}^{M} l_m$ and $N = k + 2\nu$. Since $H_0(s)$ is a ratio of even and odd polynomials then $H_0(-s) = -H_0(s)$. On the other hand, $H_1(s)$ is a ratio of odd polynomials then $H_1(-s) = H_1(s)$. When this is substituted into equation (14), we obtain $G(s) = 1$, i.e., squared magnitude characteristic of $H_0(s) + H_1(s)$ is equal to one. Thus, $H_0(s) + H_1(s)$ is an allpass function.

III. The two Channel Filter Bank

Two-channel power complementary filter bank [11] is shown in Fig. 2 is considered in this section, where $A_1(s)$ and $A_2(s)$ are two continuous time stable all-pass filters with real coefficients having no common poles.

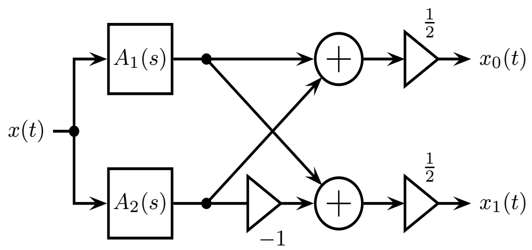

Fig. 2. The system of two-channel power complementary filter bank.

It is interesting to note that only continuous-time filters of odd degree can be realized as all-pass sums.

$$
H_0(s) = \frac{X_0(s)}{X(s)} = \frac{1}{2}[A_1(s) + A_2(s)] \tag{16}
$$

and

$$
H_1(s) = \frac{X_1(s)}{X(s)} = \frac{1}{2}[A_1(s) - A_2(s)] \tag{17}
$$

The transfer functions $H_0(s)$ and $H_1(s)$ can be implemented simply by implementing all-pass networks $A_1(s)$ and $A_2(s)$.

A. Approximation

The squared magnitude of the transfer function of the proposed n-th degree with single pair of zeros ($M = 1$ and $l_1 = 1$) at $\pm j\omega_1$ is:

$$
|H(j\omega)|^2 = \frac{1}{1 + \omega^{2k}\left(\dfrac{\omega_1^2 - \omega^2}{1 - \omega^2\omega_1^2}\right)^2} \tag{18}
$$

where $\omega_1 < 1$ determined minimum stop-band attenuation.

Performing analytic continuation $\omega = -js$, equation (18) gets form

$$
H(s)H(-s) = \frac{(1 + s^2\omega_1^2)^2}{(1 + \omega_1^2 s^2)^2 + (-1)^k s^{2k}(s^2 + \omega_1^2)^2}, \tag{19}
$$

or in simpler form, $H(s)$ can be written as

$$
H(s)H(-s) = \frac{(\omega_1^2 s^2 + 1)^2}{s^{2N} + d_1^{2(N-1)} + \cdots + d_1 s + 1} \tag{20}
$$

where

$$
d_i = \begin{cases}
(-1)^k \binom{2}{i} (\omega_1^2)^i, & i = 0, \ldots, k-1, \\
\binom{2}{i} (\omega_1^2)^i + \binom{2}{i-k} (\omega_1^2)^{2-i+k}, & i = k, \ldots, 2, \\
\binom{2}{i-k} (\omega_1^2)^{2-i+k}, & i = 3, \ldots, N
\end{cases}
$$

If $k \geq 4$, then $d_i = 0$, for $i = 4, \ldots, k$. The poles of $H(s)$ are the poles of $H(-s)$, reflected about origin. Since the desired filter function must have all poles in the left half of the s-plane, we must associate the left half plane poles of $H(s)H(-s)$ with $H(s)$. Unknown parameter ω_1 to be determined so that the minimum attenuation in the stop-band has specified value R_s. This can be done by solving a single nonlinear equation in one unknown.

B. An example

For example, by setting $k = 3$, $M = 1$, $l_1 = 1$ and $\omega_1 = 0.551017$ the order of the filter is $N = 5$, and minimum stopband attenuation is $R_s = 40$ dB. The factored form of transfer functions of the two channel filter bank in the s-plane, which is designed by using proposed method, is

$$
H_0(s) = \frac{0.303617 s^2 + 1}{(s+1)(s^2 + 0.472822 s + 1)(s^2 + 1.472822 s + 1)}
$$

and the analogue highpass prototype is

$$
H_1(s) = \frac{s^3(s^2 + 0.303617)}{(s+1)(s^2 + 0.472822 s + 1)(s^2 + 1.472822 s + 1)}
$$

These two prototypes are all-pass complementary

$$
\begin{aligned}
A_1(s) &= \frac{s^2 - 1.472821\, s + 1}{s^2 + 1.472821\, s + 1} \\
A_2(s) &= \frac{(-s+1)(s^2 - 0.472822 s + 1)}{(s+1)(s^2 + 0.472822 s + 1)}
\end{aligned}
\tag{21}
$$

and power complementary, i.e., double complementary. In terms of the pole frequency ω_p and the pole quality factor q_p, we recognize in biquad transfer function $\omega_p = 1$ (as in the case of the Butterworth filter) and first degree coefficient is $1/q_p$.

Pole-zero plots of these two allpass functions are shown in Fig. 3. In $A_2(s)$ are included the outermost pole pair, the third outermost pole pair, and so on (see the pole pair tagged with $\times$ on the Fig. 3). The remaining pole pairs belong to $A_1(s)$.

Fig. 4 gives a comparison of new filter frequency responses ($R_s = 40$ dB and $k = 5$) with Butterworth filter frequency responses. Both filters are 7th degree.

A sample of some normalized proposed transfer functions for 3 dB maximum passband attenuation and 40 dB minimum stopband attenuation, in factored form, can be found in Table I. In this table the frequencies are normalized to passband edge frequency $\omega_c = 1$. The numerator is normalized to that the dc gain of thee system is equal to unity. The passband ripple can be calculated by using (2). Since stopband ripple is 40 dB (0.01 times) then passband ripple is 4.3432×10^{-4} dB.

TABLE I
DOUBLE COMPLEMENTARY APPROXIMATION FUNCTION FOR $R_p = 40$ dB

n	Denominator of $H(s)$	Numerator of $H(s)$
3	$(s+1)(s^2+0.9124264s+1)$	$0.0875736s^2+1$
5	$(s+1)(s^2+1.4728206s+1)(s^2+0.4728206s+1)$	$0.3036200s^2+1$
7	$(s+1)(s^2+1.7136030s+1)(s^2+1.0198032s+1)(s^2+0.3062001s+1)$	$0.4649038s^2+1$
9	$(s+1)(s^2+1.8305027s+1)(s^2+1.3690518s+1)(s^2+0.7627434s+1)(s^2+0.2241943s+1)$	$0.5714580s^2+1$

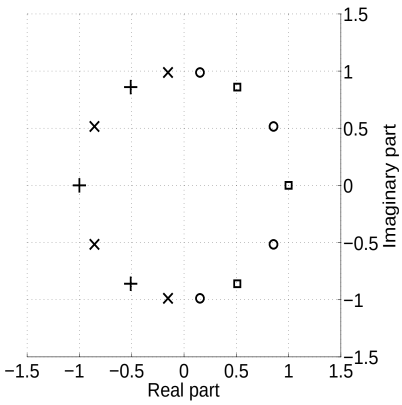

Fig. 3. Pole-zero plot of allpass functions in s-plane of the 7th-order prototype for transfer function with $R_s = 40$ dB, $M = 1$ and $l_1 = 1$. Poles and zeros of $A_1(s)$ are tagged with $\times$ and $\bigcirc$ respectively; but poles and zeros of $A_2(s)$ are tagged with $+$ and $\square$ respectively.

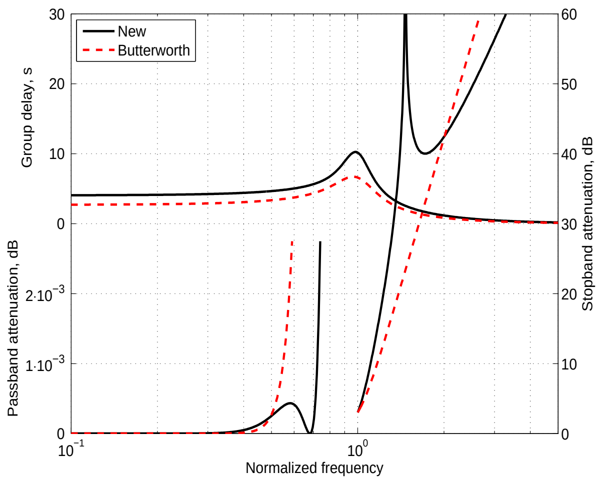

Fig. 4. Comparison of 7th degree new filter frequency responses ($R_s = 40$ dB and $k = 5$) with 7th degree Butterworth filter frequency responses.

IV. IMPLEMENTATION

In this section we discuss the design of continuous-time part of two channel hybrid filter bank based on all-pass active RC structure. All-pass transfer functions are non-minimum phase transfer function i.e., they have zeros in the right half of the complex-frequency plane.

Basically, there are two implementation manners of realizing such continuous-time allpass filter circuits. One alternative is the passive implementation, consisting of only passive components like capacitors and inductors. A number of passive circuit topologies exist, which can be used for this purpose, for instance the Lattice or T-section filters. The other alternative is the active implementation, consisting of active devices like operational amplifiers as well. Through the application of active components, it is possible to omit the bulky and costly inductor components, as well as providing more freedom in the shaping of the filter characteristic.

The active RC realization by cascading first and second order section (biquad) is proposed in this paper. The biquad can be realized with single or more operational amplifiers. The single amplifier lowpass and highpass filter will be discussed.

The transfer function of an even degree is not suitable for complementary decomposition because its allpass transfer functions have complex coefficients.

The single amplifier all-pass network first degree and the Delyiannis second degree all-pass circuit are given on the figure 5(a) and 5(b), respectively [12, chap. 4]. It is assumed that the ideal operational amplifiers are used.

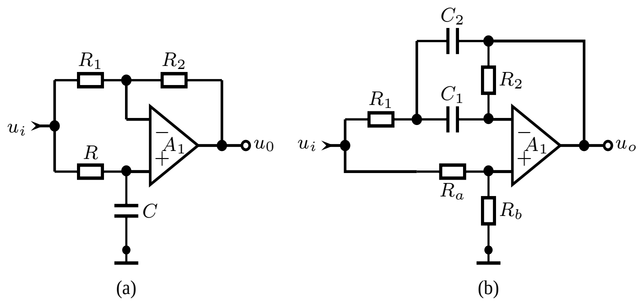

Fig. 5. (a) The single amplifier all-pass network first degree and (b) the all-pass network second degree.

The circuit on the figure 5(a) is referred to as the grounded capacitor allpass network. For $R_1 = R_2$ transfer function of first degree allpass network is

$$H(s) = -\frac{s - \dfrac{1}{RC}}{s + \dfrac{1}{RC}}$$

Transfer function of allpass network second degree, assuming ideal operational amplifier, $C_1 = C_2 = C$ is as follows

$$H(s) = h_o \frac{s^2 - \left[\dfrac{R_a}{R_b}\dfrac{1}{R_1 C} - \dfrac{2}{R_2 C}\right]s + \dfrac{1}{R_1 R_2 C^2}}{s^3 + \dfrac{2}{R_2 C}s + \dfrac{1}{R_1 R_2 C^2}} \qquad (22)$$

where $h_o = R_b/(R_a + R_b)$. Equating coefficients of equal powers of s in Eqs. (21) and (22) we can obtain the following component values

$$R_1 = \frac{1}{2Cq_p}, \quad R_2 = \frac{2q_p}{C}, \quad \frac{R_a}{R_b} = 4\frac{R_1}{R_2} \qquad (23)$$

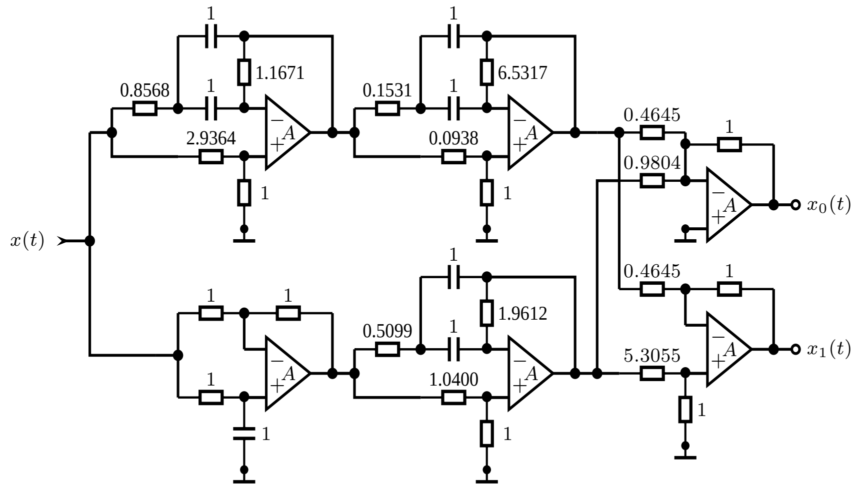

Fig. 6. Implementation of seven degree continuous-time part of two channel hybrid filter bank.

From Fig. 5, we have the implementation of the two channel continuous-time filter bank with component values shown in Fig. 6. Pole-zero plot is on the Fig. 3 given. The first all-pass filter $A_1(s)$ is realized by two second order sections shown in figure 5(b) and placed in cascade. The second all-pass filter $A_2(s)$ is realized by one first order section shown in figure 5(a) and one second order section placed in cascade.

The standard inverting summing amplifier is used for combining two signals. The differential amplifier circuit is used as subtractor. In these circuits, input signals are scaled to the desired values by selecting appropriate values for the resistors.

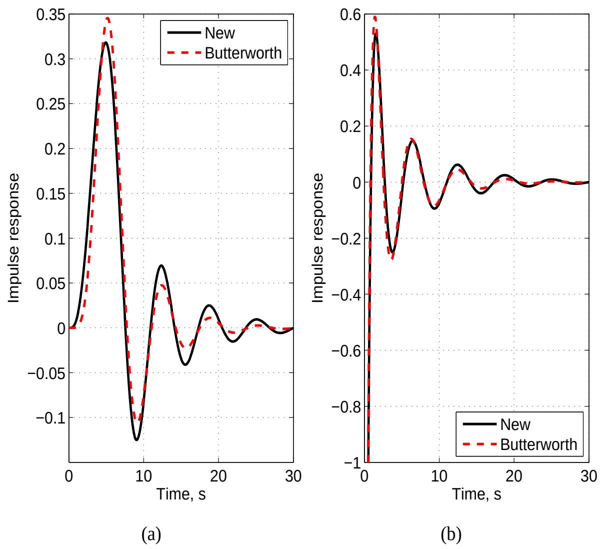

Fig. 7. Impulse responses for $N = 7$. (a) Lowpass filter, (b) highpass filter

The lowpass and highpass impulse responses for a seventh-order conventional Butterworth and proposed filter can be seen in Fig. 7. As can be seen both impulse response are very similar. In connection with Figure 4 new filter has better frequency responses in comparison with the Butterworth filter.

V. Conclusion

A new class of continuous-time filter structures has been presented, which can be used for efficient implementation of the hybrid filter bank. The conditions required to be satisfied by the transfer functions, so as to be implemented as a parallel connection of two all-pass filters, are listed. If filter degree is odd, allpas functions have real coefficients, but for even filter degree the allpass function involves in the implementation the complex coefficients. The efficiency of the proposed design has been demonstrated by means of an example.

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References

[1] S. R. Velazquez, T. Q. Nguyen, and S. R. Broadstone, "Design of hybrid filter banks for analog/digital conversion," *IEEE Trans. on Signal Processing*, vol. 46, no. 4, pp. 956–967, Apr. 1998.

[2] B. Rumberg and D. Graham, "A low-power and high-precision programmable analog filter bank," *Circuits and Systems II: Express Briefs, IEEE Transactions on*, vol. 59, no. 4, pp. 234–238, Apr. 2012.

[3] A. Kammoun, C. Lelandais-Perrault, and M. Debbah, "SNR efficient approach for the design of hybrid filter bank A/D converters," in *Acoustics, Speech and Signal Processing (ICASSP), 2013 IEEE International Conference on*, May 2013, pp. 2716–2720.

[4] N. Stamenković and V. Stojanović, "The design of two channel IIR QMF bank directly from analog prototype," *Digital Signal Processing*, vol. 98, no. 7, pp. 961–972, July 2011.

[5] I. W. Selesnick, "Lowpass filters realizable as allpass sums: design via a new at delay filter," *IEEE Trans. on Circuits and Systems Part II*, vol. 46, no. 1, pp. 40–50, Jan. 1999.

[6] S. Koshita, M. Abe, and M. Kawamata, "State-space analysis of power complementary analog filters," in *Circuits and Systems, 2007. ISCAS 2007. IEEE International Symposium on*, May 2007, pp. 2814–2817.

[7] J.-H. Lee and Y.-H. Yang, "Design of two-channel linear-phase QMF banks based on real IIR all-pass filters," *Vision, Image and Signal Processing, IEE Proceedings*, vol. 150, no. 5, pp. 331–8–, Oct. 2003.

[8] M. D. Lutovac, D. V. Tosic, and B. L. Evans, *Filter Design for Signal Processing using Matlab and Matematica*. New Jersey, USA: Prentice Hall, 2001.

[9] P. Vaidyanathan, "Some properties of iir power-symmetric filters," in *Acoustics, Speech and Signal Processing, 2007. ICASSP 2007. IEEE International Conference on*, vol. 3, Apr. 2007, pp. III–1449–III–1452.

[10] D. Živaljević and V. Stojanović, "Transitional Butter-Elliptic filter suitable for complementary decomposition," *Inverse Problems in Science and Engineering*, vol. 20, no. 1, pp. 117–125, Jan. 2012.

[11] H. Lollmann and P. Vary, "Design of iir qmf banks with near-perfect reconstruction and low complexity," in *Acoustics, Speech and Signal Processing, 2008. ICASSP 2008. IEEE International Conference on*, Mar. 2008, pp. 3521–3524.

[12] T. Deliyannis, Y. Sun, and J. K. Fidler, *Continuous-Time Active Filter Design*. Boca Raton: CRC Press LLC, 1999.