Level Crossing Rate of Macrodiversity System Operating over Gamma Shadowed Rician Fading Channel

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***Abstract*—Macrodiversity system with macrodiversity selection combining (SC) receiver and two microdiversity SC receivers is considered. Received signal is affected simultaneously to Gamma long term fading and Rician short term fading resulting in system performance degradation. Macrodiversity SC receiver reduces Gamma shadowing effects and microdiversity SC receivers reduce multipath fading effects on bit error probability. Closed form expression for average level crossing rate of macrodiversity SC receiver output signal envelope is evaluated. Numerical results are presented graphically to show the influence of Gamma long term fading severity, shadowing correlation and Rician factor on average level crossing rate.**

Keywords- fading channel; Gamma shadowed; Rician factor; selection combining (SC).

I. INTRODUCTION

Large scale fading and small scale fading degrade system performance and limit channel capacity wireless mobile communication. Reflections and refractions cause multipath propagation resulting in signal envelope variation and large obstacles cause shadowing resulting in signal envelope average power variation. Received signal experiences simultaneously long term fading and short term fading resulting in system performance degradation. There are several statistical models describing signal envelope variation in multipath fading channels depend on existence line-of-sight (LOS) components, nonlinearity of propagation channel, the number of clusters in propagation environment and variation of signal envelope power. The most frequently used statistical models which can be used to describe multipath fading are Rayleigh, Rician, Nakagami-m, Weibull, α-μ and Nakagami-q distributions. Rayleigh distributions can be used to describe small scale signal envelope variation in linear non line-of-sight multipath fading environments. In line-of-sight short term fading channels signal envelope variation can be described by using Rician distribution. Nakagami-m distribution describes multipath fading in environments with multipath scattering with large delay-time spreads and different clusters of reflected waves, while Weibull and α-μ distribution describe small scale fading in nonlinear channels. Large scale fading can be described by using log-normal distribution or Gamma distribution. When log-normal distribution describes large scale signal envelope average power, the expression for probability density function of macrodiversity receiver output signal can not be obtained in closed form. Theoretical and measured results are shown that Gamma distribution which is analytically better tractable, closely approximates the log-normal distribution in wide range of propagation conditions, and has a good fit to experimental details. When multipath fading is superimposed on shadowing, the instantaneous composite multipath/shadowed signal should be analyzed at the receiver, with Gamma distribution used for for shadowing model [1-3].

Macrodiversity system can be used to simultaneously reduce long term fading effects and short term fading effects on system performance. Macrodiversity system has macrodiversity receiver and two or more micoridiversity receivers. Macrodiversity receiver reduces long term fading effects and microdiversity receivers mitigate short term fading effects on bit error probability. The second order performance measures of wireless communication system are average level crossing rate and average fade duration. Average level crossing rate can be calculated as average value of the first derivative of random process. Average fade duration is equal to ratio of outage probability and average level crossing rate. Outage probability is defined as probability that output signal envelope falls below outage threshold [4].

There are several combining techniques which can be used to reduce long term fading effects and short term fading effects on average level crossing rate and average fade duration. The most frequently used combining techniques are maximal ratio combining (MRC), equal gain combining (EGC), selection combining (SC) and switch and stay combining (SSC). The MRC receiver enables the best results and it has highest implementation complexity. The SC receiver provides the least complexity of realization [5].

There are more works in open technical literature considering performance analysis of macrodiversity systems [6-8]. In [6], macrodiversity SC receiver with two microdiversity MRC receivers operating over Gamma shadowed Nakagami-m multipath fading environment is analyzed. Closed form expressions for average level crossing rate and average fade duration are calculated. Second order performance measures of macrodiversity system in the presence of Gamma shadowed and Rician multipath fading are analyzed.

In this paper macrodiversity system with macrodiversity SC receiver and two microdiversity SC receivers operating over composite shadowed multipath fading channel is considered. Received signal experiences correlated Gamma long term fading and Rician short term fading. Average level crossing rate of Rician random process and SC receiver output signal envelope are calculated. These expressions are used for evaluation average level crossing rate of macrodiversity SC receiver output signal envelope. To the best author's knowledge the second order statistics of macrodiversity SC receiver with two microdiversity SC receivers in the presence Gamma large scale fading and Rician small scale fading is not reported in open technical literature. Obtained results can be used in performance analysis of wireless communication system when received signal is subjected simultaneously to multipath fading and shadowing.

II. RICIAN RANDOM PROCESS AVERAGE LEVEL CROSSING RATE

Rician distribution can be used to describe large scale signal envelope variation in line-of-sight multipath fading channels with one cluster. Squared Rician random variable, x, is equal to sum of two squared independent Gaussian random variables with the same variance:

$$x^2 = x_1^2 + x_2^2, \quad (1)$$

where x_1 and x_2 are independent Gaussian random variable with the same variance σ^2. The first derivation of Rician random variable x is

$$\dot{x} = \frac{1}{x}\left(x_1\dot{x}_1 + x_2\dot{x}_2\right). \quad (2)$$

The first derivative of Gaussian random variable is Gaussian random variable. Thus $\dot{x}_1$ and $\dot{x}_2$ are zero-mean Gaussian random variables. Linear transform of Gaussian random variables is Gaussian random variable. Therefore, $\dot{x}$ follow Gaussian distribution. Mean of $\dot{x}$ is

$$\overline{\dot{x}} = \frac{1}{x}\left(x_1\overline{\dot{x}_1} + x_2\overline{\dot{x}_2}\right) = 0, \quad (3)$$

since $\overline{\dot{x}_1} = \overline{\dot{x}_2} = 0$.

The variance of the first derivate of Rician random variable is

$$\sigma_{\dot{x}}^2 = \frac{1}{x^2}\left(x_1^2\sigma_{\dot{x}_1}^2 + x_2^2\sigma_{\dot{x}_2}^2\right), \quad (4)$$

where [10]:

$$\sigma_{\dot{x}_1}^2 = \sigma_{\dot{x}_2}^2 = \pi^2 f_m^2 2\sigma^2, \quad (5)$$

where f_m is maximal Doppler frequency. After substituting (5) in (4), the variance of the first derivative of Rician random variable becomes

$$\sigma_{\dot{x}}^2 = \frac{1}{x^2}\pi^2 f_m^2 2\sigma^2\left(x_1^2 + x_2^2\right) = 2\pi^2 f_m^2\sigma^2. \quad (6)$$

The joint probability density function of Rician random variable and the first derivative of Rician random variable is

$$p_{x\dot{x}}\left(x\dot{x}\right) = p_x\left(x\right)p_{\dot{x}}\left(\dot{x}\right), \quad (7)$$

where $p_x(x)$ is probability density function of x:

$$p_x\left(x\right) = \frac{2\left(1+k\right)}{e^k\Omega}e^{-\frac{(1+k)x^2}{\Omega}}I_0\left(2\sqrt{\frac{k\left(1+k\right)}{\Omega}}x\right), \quad (8)$$

where k is Rician factor. Rician factor can be evaluated as ratio of dominant component power to scattering component power. The level crossing rate of Rician random process can be calculated as average value of the first derivative of Rician random variable:

$$\begin{aligned} N_x &= \int_0^\infty d\dot{x}\cdot\dot{x}\cdot p_{x\dot{x}}\left(x\dot{x}\right) = p\left(x\right)\frac{1}{\sqrt{2\pi}}\sigma_{\dot{x}} = \\ &= \frac{f_m\sqrt{2}\left(1+k\right)}{e^k\Omega^{\frac{1}{2}}}e^{-\frac{(1+k)x^2}{\Omega}}I_0\left(2\sqrt{\frac{k\left(1+k\right)}{\Omega}}x\right) \end{aligned}. \quad (9)$$

Expression for level crossing rate can be used for evaluation average fade duration of wireless communication system operating over Rician multipath fading channel. The cumulative distribution function of Rician distribution is

$$F_x(x)=\int_0^x p_x(t)\,dt = \\ =\frac{1}{e^k}\sum_{i_1=0}^{\infty}\frac{k^{i_1}}{(i_1!)^2}\gamma\left(i_1+1,\frac{(k+1)x^2}{\Omega}\right). \tag{10}$$

Obtained expression for cumulative distribution function of Rician random variable can be used in performance analysis of wireless communication system in the presence Rician short term fading.

The average level crossing rate of dual SC receiver operating over independent identical Rician multipath fading output signal envelope is

$$N_x = 2F_{x_1}(x)N_{x_1}, \tag{11}$$

where N_{x1} is given with (9) and $F_{x1}(x)$ is given with (10). The expression can be used for calculation average fade duration of communication mobile system with SC reception operating over Rician multipath fading channel.

III. Level Crossing Rate of Macrodiversity SC Receiver Output Signal Envelope

Macrodiversity system with macrodiversity SC receiver and two microdiversity SC receivers operating over composite Gamma shadowed Rician multipath fading environment is considered. Microdiversity SC receiver is provided by using multiple antennas at base station resulting in reduction of Rician multipath fading effects and macrodiversity SC receiver uses signals from two or more base stations resulting in reduction Gamma long term effects on system performance. Signal envelopes at output of microdiversity SC receivers are denoted with x_1 and x_2. Macrodiversity SC receiver output signal envelope is denoted with x.

Signal envelope average power at inputs in microdiversity SC receivers are denoted with Ω_1 and Ω_2. Random variable Ω_1 and Ω_2 follow correlated Gamma distribution:

$$p_{\Omega_1\Omega_2}(\Omega_1\Omega_2)=\frac{1}{\Gamma(c)(1-\rho^2)\rho^{\frac{c-1}{2}}\Omega_0^{c+1}} \\ \times\sum_{i_2=0}^{\infty}\left(\frac{\rho}{\Omega_0(1-\rho^2)}\right)^{2i_2+c-1}\frac{1}{i_2!\Gamma(i_2+c)} \\ \times\Omega_1^{i_2+c-1}\Omega_2^{i_2+c-1}e^{-\frac{\Omega_1+\Omega_2}{\Omega_0(1-\rho^2)}}, \tag{12}$$

where c is order of Gamma distribution, ρ is correlation coefficient and Ω_0 is average power of Ω_1 and Ω_2.

Macrodiversity SC receiver selects microdiversity SC receiver with higher signal envelope average power at inputs to provide service to users. Therefore, average level crossing rate of macrodiversity SC receiver output signal envelope is

$$N_x=\int_0^\infty d\Omega_1\int_0^{\Omega_1} d\Omega_2 N_{x_1/\Omega_1}p_{\Omega_1\Omega_2}(\Omega_1\Omega_2)+ \\ +\int_0^\infty d\Omega_2\int_0^{\Omega_2} d\Omega_1 N_{x_2/\Omega_2}p_{\Omega_1\Omega_2}(\Omega_1\Omega_2), \\ =2\int_0^\infty d\Omega_1\int_0^{\Omega_1} d\Omega_2 N_{x_1/\Omega_1}p_{\Omega_1\Omega_2}(\Omega_1\Omega_2) \tag{13}$$

where $N_{x1/\Omega1}$ and $N_{x2/\Omega2}$ is given with (11). By substituting (9), (10) and (12) in (13), the expression for average level crossing rate becomes

$$N_x=\frac{4\sqrt{2}f_m(1+k)}{e^{2k}\Gamma(c)(1-\rho^2)\rho^{\frac{c-1}{2}}\Omega_0} \\ \times\sum_{i_1=0}^{\infty}\frac{k^{i_1}}{(i_1!)^2}\sum_{i_3=0}^{\infty}\frac{(k(1+k))^{i_3}x^{2i_3}}{(i_3!)^2} \\ \times\sum_{i_2=0}^{\infty}\left(\frac{\rho}{\Omega_0(1-\rho^2)}\right)^{c+2i_2-1}\frac{(\Omega_0(1-\rho^2))^{i_2+c}}{i_2!\Gamma(i_2+c)} \\ \times\int_0^\infty d\Omega_1\Omega_1^{i_2-i_3+c-\frac{3}{2}}e^{-\frac{\Omega_1}{\Omega_0(1-\rho^2)}-\frac{(1+k)x^2}{\Omega_1}} \\ \times\gamma\left(i_1+1,\frac{(k+1)x^2}{\Omega_1}\right)\gamma\left(i_2+1,\frac{\Omega_1}{\Omega_0(1-\rho^2)}\right)= \\ =\frac{4\sqrt{2}f_m(1+k)}{e^{2k}\Gamma(c)(1-\rho^2)\rho^{\frac{c-1}{2}}\Omega_0}\sum_{i_1=0}^{\infty}\frac{k^{i_1}}{(i_1!)^2}\sum_{i_3=0}^{\infty}\frac{x^{2i_3}}{(i_3!)^2} \\ \times\sum_{i_2=0}^{\infty}\left(\frac{\rho}{\Omega_0(1-\rho^2)}\right)^{c+2i_2-1}\frac{(\Omega_0(1-\rho^2))^{i_2+c}}{i_2!\Gamma(i_2+c)}\times I, \tag{14}$$

where $\gamma(n,x)$ [9] is incomplete Gamma function and I is

$$I=\int_0^\infty d\Omega_1\Omega_1^{i_2-i_3+c-\frac{3}{2}}e^{-\frac{\Omega_1}{\Omega_0(1-\rho^2)}-\frac{(1+k)x^2}{\Omega_1}} \\ \left(\frac{1}{i_1+1}\left(\frac{(k+1)x^2}{\Omega_1}\right)^{i_1+1}\right. \\ \left.\times e^{-\frac{(k+1)x^2}{\Omega_1}}\sum_{j_1=0}^{\infty}\frac{1}{(i_1+2)_{(j_1)}}\left(\frac{(k+1)x^2}{\Omega_1}\right)^{j_1}\right)$$

$$\times\left(\frac{1}{i_2+1}\left(\frac{\Omega_1}{\Omega_0\left(1-\rho^2\right)}\right)^{i_2+1}\right.$$

$$\left.\times e^{-\frac{\Omega_1}{\Omega_0\left(1-\rho^2\right)}}\sum_{j_2=0}^{\infty}\frac{1}{\left(i_2+2\right)_{(j_2)}}\left(\frac{\Omega_1}{\Omega_0\left(1-\rho^2\right)}\right)^{j_2}\right)= \tag{15}$$

$$=I_1-I_2-I_3+I_4$$

where $(a)_{(n)}$ denoting the Pocchammer symbol.

The integral I_1 can be solved as:

$$I_1=\Gamma\left(i_1+1\right)\Gamma\left(i_2+1\right)$$
$$\times\left(\left(k+1\right)x^2\Omega_0\left(1-\rho^2\right)\right)^{\frac{i_2}{2}-\frac{i_3}{2}+\frac{c}{2}-\frac{1}{4}}. \tag{16}$$
$$\times K_{i_2-i_3+c-\frac{1}{4}}\left(2\sqrt{\frac{\left(k+1\right)x^2}{\Omega_0\left(1-\rho^2\right)}}\right)$$

The integral I_2 can be solved as:

$$I_2=\frac{\Gamma\left(i_1+1\right)}{i_2+1}\sum_{j_2=0}^{\infty}\frac{1}{\left(i_2+2\right)_{(j_2)}}$$
$$\times\frac{1}{\left(\Omega_0\left(1-\rho^2\right)\right)^{i_2+j_2+1}} \tag{17}$$
$$\times\left(\frac{\left(k+1\right)x^2\Omega_0\left(1-\rho^2\right)}{2}\right)^{i_2-\frac{i_3}{2}+\frac{j_2}{2}+\frac{c}{2}+\frac{1}{4}}$$
$$\times K_{2i_2-i_3+j_2+c+\frac{1}{2}}\left(2\sqrt{\frac{2\left(k+1\right)x^2}{\Omega_0\left(1-\rho^2\right)}}\right).$$

The integral I_3 can be solved as:

$$I_3=\frac{\Gamma\left(i_2+1\right)}{i_1+1}\sum_{j_1=0}^{\infty}\frac{\left(\left(k+1\right)x^2\right)^{i_1+j_1+1}}{\left(i_1+2\right)_{(j_1)}}$$
$$\times\left(2\left(k+1\right)x^2\Omega_0\left(1-\rho^2\right)\right)^{\frac{i_2}{2}-\frac{i_3}{2}-\frac{j_1}{2}+\frac{c}{2}-\frac{3}{4}}. \tag{18}$$
$$\times K_{i_2-i_3-j_1+c-\frac{3}{2}}\left(2\sqrt{\frac{2\left(k+1\right)x^2}{\Omega_0\left(1-\rho^2\right)}}\right)$$

The integral I_4 can be solved as:

$$I_4=\frac{1}{i_1+1}\sum_{j_1=0}^{\infty}\frac{\left(\left(k+1\right)x^2\right)^{i_1+j_1+1}}{\left(i_1+2\right)_{(j_1)}}$$
$$\times\frac{1}{i_2+1}\sum_{j_2=0}^{\infty}\frac{1}{\left(i_2+2\right)_{(j_2)}}\frac{1}{\left(\Omega_0\left(1-\rho^2\right)\right)^{i_2+j_2+1}}, \tag{19}$$
$$\times\left(\left(k+1\right)x^2\Omega_0\left(1-\rho^2\right)\right)^{-\frac{i_1}{2}+i_2-\frac{i_3}{2}-\frac{j_1}{2}+\frac{j_2}{2}+\frac{c}{2}-\frac{1}{4}}$$
$$\times K_{-i_1+2i_2-i_3-j_1+j_2+c-\frac{1}{2}}\left(2\sqrt{\frac{4\left(k+1\right)x^2}{\Omega_0\left(1-\rho^2\right)}}\right)$$

where $K_n(x)$ is modified Bessel function of the second kind [9], order n and argument x.

IV. NUMERICAL RESULTS

In Figure 1 and Figure 2, normalized average level crossing rate of macrodiversity SC receiver output signal envelope is presented in the function of system parameters such are Gamma shadowing severity parameter c, correlation coefficient ρ and Rician factor K.

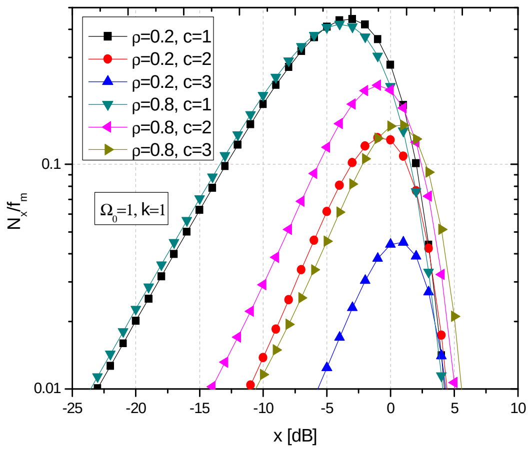

Figure 1. Level crossing rate of macrodiversity SC receiver output signal envelope for different values of Gamma shadowing severity parameter c and correlation coefficient ρ.

For lower values of SC receiver output signal envelope, average level crossing rate increases as signal envelope increases and for higher values of signal envelope, level crossing rate decreases as signal envelope increases.

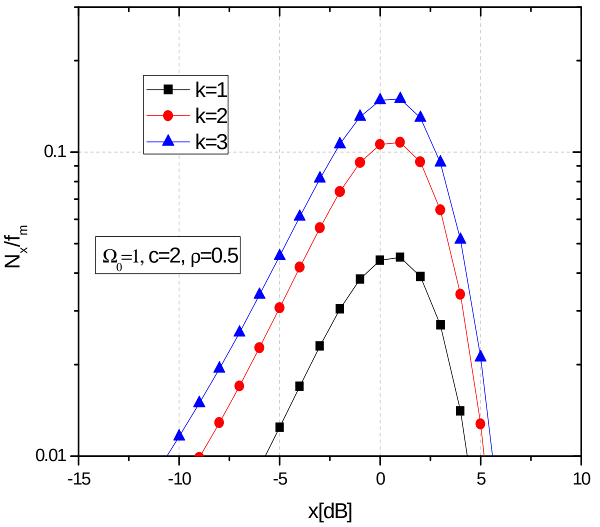

Figure 2. Level crossing rate of macrodiversity SC receiver output signal envelope for different values of Rician factor k.

V. CONCLUSION

SC macrodiversity system with dual SC microdiversity operating over composite shadowed Ricean/Gamma fading channel is considered. Expressions for LCR of the macrodiversity output are presented in closed-form. LCR dependence on observed szstem parameters, such are Gamma shadowing severity parameter c, correlation coefficient ρ and Rician factor K has also been graphicaly presented and discussed.

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